METHOD AND APPARATUS FOR REDUCING THE CREST FACTOR OF A SIGNAL

Background

The present invention relates to a method and to an apparatus for reducing the crest factor (the ratio of peak to average value) of a signal, for example, a multi-carrier communication signal.

In recent years multi-carrier communication systems have been widely used in particular for x DSL communication systems (Digital Subscriber Line) like ADSL (Asymmetric Digital Subscriber Line) or VDSL (Very High Speed Digital Subscriber Line). Figure 1 illustrates a schematic block diagram of such a transmission system. A serial data signal a is fed to a serial/parallel converter 1 that converts the serial digital data signal a into data packets with N/2 sub-packets, N being an even number. One data packet is transmitted in parallel to an encoder 2, which assigns each sub packet to a separate carrier frequency and supplies a first digital signal vector to an inverse Fourier transformer 3, which performs an inverse Fourier transformation on this vector and generates a second digital signal vector comprising N samples of a signal to be sent. This second digital signal vector is transmitted to a parallel/serial converter 23, which supplies the elements or samples of the second digital signal vector to a digital filter 24 followed by a digital-to-analog converter 25 and a line driver 26. The generated analog transmit signal is transmitted via a channel 27, whereby noise b is added, symbolized by an adder 28. On the receiver side, the signal is equalized by an equalizer/an analog-to-digital converter 29. Then the signal is decoded by performing the reverse operations of the encoding elements 1 to 23, namely through a serial/parallel converter 30, a Fourier transformer 31, a decoder 32, a slicer 33 and a parallel/serial converter 34.

Such a communication system is for example disclosed in US 6,529,925 B1, the contact of which is incorporated by reference herein.

Since the transmit signal is composed of a plurality of different signals having different carrier frequencies and amplitudes and phases being determined

by the data signal and thus having no predetermined relationships, the amplitude of the transmit signal has approximately a Gaussian distribution. Figure 2 illustrates the probability h of the amplitude A of the transmit signal as determined by a simulation for a discrete multitone modulated transmit signal with a Fourier transform block length of 256.

Because of this Gaussian distribution, the crest factor of the signal is rather large, that is, the transmit signal has a rather high maximum amplitude value compared to the effective or average amplitude value. Since both the digital-to-analog and analog-to-digital converters, as well as the line drivers, have to be adapted to handle the whole possible amplitude range, these elements have to be defined accordingly causing additional costs and chip space. It is therefore desirable to reduce the crest factor, that is to reduce the maximum amplitude.

In principle, two different approaches are known to reduce the crest factor. A first method for reducing the maximum amplitudes disturbs the transmit signal. These methods comprise clipping methods as described for example in US patent no. 6,038,261. A second method for reducing the maximum amplitude without disturbing the signal.

In general, these methods use one or more of the carrier frequencies to modify the transmit signal in order to reduce the maximum amplitude. The carrier frequencies used for this purpose may or may not only partially be used for the actual data transmission.

One of these methods is described in the already cited US patent 6,529,925 B1. There, the Nyquist frequency is used as a single carrier frequency for correction purposes, that is, the last frequency in the inverse Fourier transform. In an ADSL signal, this frequency is not used for data transmission so that the correction does not influence the transmission capacity. However, the performance of this method is limited since only a single carrier frequency is used for correction. Furthermore, this method is not applicable to VDSL signals since the Nyquist frequency is outside the usable frequency range both for downstream and for upstream transmission.

In US 6,424,681 B1 a method for reducing the crest factor using a plurality of carrier frequencies is disclosed. These carrier frequencies are

preferably evenly distributed over the whole usable frequency range. From these carrier frequencies a normalized correction signal, a so-called kernel, is generated which has a “Dirac”-like shape, that is, a shape that comprises a single peak as far as possible. To correct a transmit signal, this correction signal is phase shifted to the position of the maximum of the transmit signal and then scaled with a suitable scaling factor depending on the maximum amplitude of the transmit signal. Then, this correction signal is subtracted from the transmit signal. This can be repeated several times to iteratively correct several maximum or peak values. For transmission systems with a great number of carrier frequencies and consequently a great number of signal values in each frame, like a VDSL transmission system, this method is difficult to realize since it needs a relatively long computation time. Furthermore, through the use of a kernel, the carrier frequencies used for the correction have to comprise both low and high frequencies which, consequently, are not usable for data transmission. The use of low carrier frequencies, on the other hand, leads to a greater loss of transmission capacity since lower carrier frequencies can be modulated with more bits than high carrier frequencies due to the lower damping.

Summary

One embodiment of the present invention provides a method and an apparatus that effectively reduce the crest factor using a limited number of carrier frequencies. Furthermore, one embodiment provides such a method and such an apparatus that are usable for VDSL transmission.

According to one embodiment of the invention for reducing the crest factor of a signal using a plurality of partial correction signals having predetermined frequencies, the following steps are carried out:

(a) determining a time position of a maximum absolute amplitude of the signal,

(b) calculating an amplitude and a phase for the respective partial correction signal depending on said maximum absolute amplitude and said time position determined in step (a),

(c) subtracting the respective partial correction signal from said signal to obtain a partially corrected signal which is used as the signal in step (a)

for the next one of the plurality of partial correction signals, and returning to step (a) for calculating an amplitude and a phase for the next partial correction signal,

said method further comprising the step of

(d) outputting the last obtained partially corrected signal as the corrected signal having the reduced crest factor.

As for each of the predetermined frequencies, i.e. carrier frequencies, an amplitude and a phase is calculated, and it is possible to use the predetermined frequencies available in an optimum manner to correct the signal.

Steps (a) to (c) may be repeated a given number of iterations to obtain even better results.

In step (b), the amplitude may be calculated according to

$$A = g \cdot (\max\{X(t) \cdot \cos(2\pi f(t - t\max))\} + \min\{X(t) \cdot \cos(2\pi f(t - t\max))\})$$

A being the amplitude, g being a predetermined factor, f being the respective predetermined frequency, t being the time, $t\max$ being said time position, $X(t)$ being said signal and $\max$ and $\min$ designate the maximum and minimum operator, respectively. The phase accordingly amounts to $2\pi f \cdot t\max$.

For discrete multitone modulation signals as mentioned in the introductory portion, the signal may be represented as a signal vector of signal values at N sampling times. Accordingly, the above formula may be reformulated basically by replacing the time by the number of the sample and replacing the frequency by a number of the frequency divided by N.

The method as described so far is suitable for signals where the signal vector does not comprise too many samples. For transmission systems like VDSL systems carrying out the described method in full would cost considerable calculation time.

Therefore, instead of performing the above method on the signal or on a vector representing the signal, one embodiment performs the method on a vector containing only a predetermined number of maximum amplitude values of the signal. This predetermined number may be significantly lower than the number

of samples in the actual signal vector, therefore saving considerable calculation time while only marginally lowering the performance. To do this, the positions of the elements of the vector with the maximum amplitude values in the original signal vector have to be stored since the final correction has to be performed on the signal itself.

Brief Description of the Drawings

The accompanying drawings are included to provide a further understanding of the present invention and are incorporated in and constitute a part of this specification. The drawings illustrate the embodiments of the present invention and together with the description serve to explain the principles of the invention. Other embodiments of the present invention and many of the intended advantages of the present invention will be readily appreciated as they become better understood by reference to the following detailed description. The elements of the drawings are not necessarily to scale relative to each other. Like reference numerals designate corresponding similar parts.

Figure 3 illustrates an embodiment of an apparatus for reducing a crest factor of a signal according to the present invention.

Figures 4A and 4B illustrates simulations of the performance of the method of the present invention for ADSL signals.

Figures 5A and 5B illustrate further simulations of the performance of the method of the present invention for ADSL signals.

Figures 6A and 6B illustrate usable frequency ranges for VDSL in a downstream and in an upstream direction, respectively.

Figures 7A and 7B illustrate simulation results for VDSL upstream.

Figures 8A and 8B show simulation results for VDSL downstream.

Figure 1 illustrates a standard multi-carrier transmission system.

Figure 2 illustrates an amplitude probability distribution for a standard multi-carrier transmission system.

Detailed Description

As already described in the introductory portion with reference to Figure 1, a transmit signal in a multitone transmission like discrete multitone

transmission comprises a number of samples derived from parallel processing of a number of bits of serial data, a data block. This transmit signal may be described as a vector

$$X^T = [x(1), x(2), \ldots, x(N)], \qquad (1)$$

N being the number of samples and $x(n)$ being the respective samples, n ranging from 1 to N. The index n thus denotes the time position of the respective sample. "T" indicates that the vector in equation (1) is written in a line instead of in a column.

One embodiment of the present invention is to determine a correction vector Xk so that the maximum absolute value or amplitude of the elements of the vector Xs with

$$Xs = X - Xk \qquad (2)$$

assumes a minimum value. The correction vector Xk is a superposition of several partial correction vectors corresponding to a number of carrier frequencies or carrier tones reserved for forming the correction vector Xs, i.e.

$$Xk = \sum_{i=1}^{Nt} Xk_i\ , \qquad (3)$$

Nt being the number of carrier frequencies reserved for correction. Xk_i denotes the i-th correction vector contribution of carrier frequency number i.

In general, the components of Xk_i may be written as

$$xk_i(k) = a_i(\mu)\cos\left(2\pi\mu \cdot \frac{k-1}{N}\right) + b_i(\mu)\sin\left(2\pi\mu\frac{k-1}{N}\right), \qquad (4)$$

wherein k is the component or sample index of the vector, ranging from 1 to N, and μ is the number of the respective carrier frequency used to form the

correction vector xk_i assuming that all the carrier frequencies used including those used for transmitting the actual information are numbered consecutively and are spaced evenly apart from each other, starting with 0. Such a numbering of carrier frequencies is usually used for example for the carrier frequencies of ADSL or VDSL transmission. Generally, equation (4) is an oscillation with the frequency determined by μ and an amplitude and phase determined by a_i and b_i.

In the following, an iterative procedure for correcting the vector X and for determining the correction vector Xk is given, comprising the following steps:

1. determining the element of the vector X having the maximum absolute amplitude $|X(k\max)|$ and its position within the vector X $k\max$, and

2. forming an auxiliary vector Xh according to

$$xh(k) = x(k)\cos\left(2\pi\mu\frac{k-k\max}{N}\right); k = 1,2,...N, \qquad (5)$$

xh being the components of Xh. Since the cosine term for $k = k\max$ is equal to 1, the auxiliary vector has the element with the same maximum absolute amplitude at the same position $k\max$ as the vector X.

3. For the carrier frequency μ, a partial correction is carried out according to:

$$x(k)_{new} = x(k)_{old} - g(\max\{xh(k)\} + \min\{xh(k)\}) \cdot 0.5 \cdot \cos\left(2\pi\mu\frac{k-k\max}{N}\right), \quad (6)$$

$$k = 1,2,...,N,$$

wherein the indices "new" and "old" indicate that the elements of the vector X are replaced by the new elements. In equation (6), max is the maximum operator yielding the maximum value of all the $xh(k)$ and min is the corresponding minimum operator. In general, the minimum will be a negative value. It should

be noted in this respect that the maximum absolute amplitude determined in step 1 may be either the maximum or the minimum value. The factor g is an appropriate converging factor which may be chosen to be 1 or may vary from iteration to iteration as explained below. The factors 0.5 and g may be drawn into a single factor.

4. also given is repeating steps 1 to 3 for all carrier frequencies μ used for correcting the signal, whereby the "new" vector X is used for the respective next carrier frequency.

5. also given is repeating steps 1 to 4 L times. The converging factor g can be chosen to decrease from iteration to iteration ensuring a better convergence.

The total correction vector Xk would then be a sum of all the corrections carried out in step 3.

The vector $X^T = [x(1), x(2), ..., x(N)]$ resulting from this method has the smallest maximum absolute amplitude possible with a correction signal consisting of the given correction carrier frequencies.

On the other hand, for vectors X having a large number of elements this algorithm needs a large realization effort since for each iteration for each carrier frequency used for correction a correction term has to be subtracted componentwise from the original vector X. For example, for a VDSL transmission vector the vector X has 8192 elements.

Therefore, a simplification of the above algorithm is needed for vectors X having a large number of elements. The general idea for simplifying the above algorithm is not to perform the algorithm on the complete vector X, but on an auxiliary vector $Xm^T = [xm(1), xm(2), ..., xm(M)]$ comprising the M elements having the largest absolute amplitudes of the vector X, M being much smaller than N. For example, for VDSL systems M may be chosen to be 32 which is considerably smaller than 8192, thus saving considerably computation time. Since the correction itself has to be performed on the whole signal, that is on the vector X, a further auxiliary vector

$Pm^T = [pm(1), pm(2), ..., pm(M)]$ is needed for storing the positions of the elements of the vector Xm within the vector X, i.e. $xm(k) = x(pm(k))$.

An algorithm for determining the vectors Xm and Pm will be given later.

In the following, it will be shown how the algorithm described above has to be performed using the vector Xm. The following steps have to be carried out corresponding to the respective steps of the algorithm already explained:

1. the position $k\max$ of the element of the vector Xm having the largest absolute amplitude or value $|xm(k\max)|$ is determined and

2. an auxiliary vector Xmh according to

$$xmh(k) = xm(k) \cdot \cos\left(2\pi\mu \frac{pm(k) - pm(k\max)}{N}\right); k = 1,2,...,M \quad (7)$$

is calculated, wherein the xmh are the components of the vector Xmh. Thus, through the use of the vector Pm within the cosine term, the cosine term assumes the "correct" values for the elements of the vector Xm that correspond to the values of the correction finally performed on the whole signal X.

3. A partial correction corresponding to the one for the full vector X is carried out:

$$xm(k)_{new} = xm(k)_{old} - g \cdot (\max\{xmh(k)\} + \min\{xmh(k)\}) \cdot 0.5 \cdot \quad (8)$$
$$\underline{\cdot \cos\left(2\pi\mu \frac{pm(k) - pm(k\max)}{N}\right); k = 1,2,...,M \,.}$$

After performing the algorithm the correction vector Xk for the signal vector, X has to be calculated. To this end, it is helpful to store for each partial correction and for each frequency μ the correction amplitude

$$\Delta u(i, j) = g \cdot (\max\{xmh(k)\} + \min\{xmh(k)\}) \cdot 0.5, \quad (9)$$

and the corresponding phase

$$\Delta p(i,j) = pm(k_{\max}), \qquad (10)$$

Wherein i is again the number of the carrier frequency as in equation (4) and j is the number of iteration.

4. Finally steps 1 to 3 are repeated for all carrier frequencies μ used for the correction signal or vector, and

5. steps are repeated 1 to 4 L times, possibly with decreasing converging parameter g.

At the end of this procedure, an auxiliary vector Xm having a minimal maximum absolute amplitude is obtained. From the stored amplitude and phase values $\Delta u(i,j)$ and $\Delta p(i,j)$, the amplitudes and phases for the final correction vector Xk with N components can be calculated. Each partial correction vector for a single correction carrier frequency μ can be calculated according to equation (4). The respective correction amplitudes $a_i(\mu)$ and $b_i(\mu)$ can be calculated as follows:

$$a_i(\mu) = \sum_j \Delta u(i,j)\cos\left(2\pi\Delta p(i,j)\cdot\frac{\mu}{N}\right)$$

$$b_i(\mu) = \sum_j \Delta u(i,j)\sin\left(2\pi\Delta p(i,j)\frac{\mu}{N}\right). \qquad (11)$$

Alternatively, the amplitudes $a_i(\mu)$ and $b_i(\mu)$ may be computed iteratively in step 3 of the algorithm given above, so that the Δu and Δp do not have to be stored. In this case, in step 3, the following calculations have to be performed:

$$a_i(\mu)_{new} = a_i(\mu)_{old} + \Delta u(i,j)\cdot\cos\left(2\pi\mu\frac{\Delta p(i,j)}{N}\right) \qquad (12)$$

$$b_i(\mu)_{new} = b_i(\mu)_{old} + \Delta u(i,j) \cdot \sin\left(2\pi\mu \frac{\Delta p(i,j)}{N}\right).$$

The correction vector is composed of cosine and sine values weighed with respective amplitude values. The cosine and sine values can be read from a sine table or a cosine table. One table is sufficient for the cosine and the sine values since its two functions only differ in the phase, thatis the respective address of the table read out has to be adapted. The use of such a sine table makes the algorithm faster compared to explicitly calculating the sine or cosine values each time.

A reduction of the values to be stored in such a table can be obtained if the sine or cosine value is calculated as an interpolation, for example a linear interpolation, between stored sine values. It has turned out that the storage of 32 sine values of a quarter period is sufficient. The remaining three quarters of the period of the sine and cosine functions can be calculated using the symmetry of these functions.

It is possible to write the partial correction vector Xk_i also as

$$xk_i(k) = c_i(\mu)\cos\left(2\pi\mu \frac{k-1}{N} + \varphi_i(\mu)\right) \tag{13}$$

with

$$c_i(\mu) = \sqrt{a_i^2(\mu) + b_i^2(\mu)}; \qquad \varphi_i(\mu) = \arctan\left(\frac{b_i(\mu)}{a_i(\mu)}\right), \tag{14}$$

arctan being the arcus tangens operator.

In this case, only a single sine value has to be calculated or read out from the sine table for each partial correction vector with the carrier frequency μ. The values for $c_i(\mu)$ and $\varphi_i(\mu)$ from equation (14) may also be calculated using the known Cordic algorithm. This algorithm is used for calculating amplitude of a phase of a complex number when its real end imaginary part is given. As a real part a_i and as imaginary part b_i can be taken. The Cordic algorithm is an iterative algorithm that uses only additions and subtractions as

well as the sign function for determining the sine of a number. For performing the algorithm, L arcus tangens values have to be stored, L being the number of iterations of the Cordic algorithm. After carrying out the Cordic algorithm the amplitude of the respective complex number which results from the algorithm is enlarged by a fixed factor dependent on L. Therefore, it is necessary to divide this value by this factor. To be able to omit these divisions, the values of the sine table may be divided by the factor in advance.

A further reduction of computing effort can be obtained if the stored values of the sine table, for example 32 values, are multiplied with the respective amplitude of the correction carrier frequency in advance and stored in an intermediate storage. For computing the partial correction vector this intermediate storage would simply have to be addressed. No further multiplication would be necessary.

The realization effort can be further reduced significantly by choosing the carrier frequencies used for correction in an appropriate manner. If, for the representation of equation (13), the carrier frequency is chosen as

$$\mu = 2^{\ell} \cdot v \qquad (15)$$

and the number of elements in the vector X is

$$N = 2^{n} \qquad (16)$$

which is generally the case for system using inverse fast Fourier transform like the system described in the introductory portion, equation (14) transforms to

$$xk_i(k) = c_i(\mu)\cos\left(2\pi 2^{\ell} \cdot v \cdot \frac{k-1}{2^{n}} + \varphi_i(\mu)\right) \qquad (17)$$

or

$$xk_i(k) = c_i(\mu)\cos\left(2\pi \cdot v \cdot \frac{k-1}{2^{n-\ell}} + \varphi_{\mu}\right). \qquad (18)$$

As can be easily seen, the partial correction vector Xk_i is periodic with a period of $2^{n-\ell}$. If the different carrier frequencies used for correction differ only in the value v, the resulting correction vector Xk only has to be calculated for the first $2^{n-\ell}$ values. The whole correction vector is then obtained through periodic continuation.

For the simplified algorithm, as stated above, the auxiliary vector Xm is needed containing the M values of the vector X having the largest absolute amplitude values. A possible algorithm for obtaining the vector Xm and the vector Pm comprises the following steps:

1. The vector Xm is initialized to contain the M last elements of the vector X, that is,

$$xm(k) = x(N - M + k); \qquad k = 1,2,\ldots,M\,. \tag{19}$$

2. The vector Pm is initialized accordingly, that is,

$$pm(k) = N - M + k; \qquad k = 1,2,\ldots,M\,. \tag{20}$$

3. A counter λ is set to 0: $\lambda = 0$.

4. The element of the vector Xm having the smallest absolute value is determined:

$$x\min = \min\{|xm(k)|\} \tag{21}$$

$x\min$ being that minimum value.

5. The corresponding position $k\min$ is determined:

$$k\min = \mathit{Position\ of}\ \min\{|xm(k)|\};\ \text{that is,} \tag{22}$$

$$|xm(k\min)| = \min\{|xm(k)|\}.$$

6. The counter λ is incremented: $\lambda = \lambda + 1$.

7. The element of the vector X designated by the counter λ is compared with $x\min$; steps 6 and 7 are repeated until

$$|x(\lambda)| > x\min\,.$$

8. When $|x(\lambda)| > x\min$ is fulfilled, the minimal element of the vector Xm is replaced by the element of the vector X designated by the counter λ, and the corresponding element of the vector Pm is replaced by λ, that is,

$$xm(k\min) = x(\lambda)$$

$$pm(k\min) = \lambda.$$

9. The procedure is continued at step 6 until λ has reached the value $N - M$

When this procedure is completed, the vector Xm contains the M values having the largest absolute amplitudes of the vector X, and the vector Pm contains the corresponding positions.

The time needed for performing this algorithm depends on the arbitrary values of the starting vector for Xm. The more large values this vector contains when starting the procedure, the less often the contents of the vector Xm has to be overwritten and the minimum element of the vector Xm be determined. Consequently, through a pre-sorting of the vector X it is possible to optimize this algorithm.

Figure 3 illustrates an apparatus suitable for carrying out the method of the present invention corresponding to the algorithms described above. A data signal a is supplied to a serial-to-parallel converter 1 and modulated onto a number of carrier frequencies wherein a predetermined number of carrier frequencies are not used for transmitting the data, but for building the correction signal as described above. On the thus generated signal, an inverse Fourier transform is performed in element 3, and the data is supplied to a parallel-to-serial converter 23, which is generally used for serially outputting the corresponding signal vector. Up to this point, the apparatus corresponds to the one already described with reference to Figure 1 above, that is, in converter 23 the vector X is stored. The vector X is transmitted to means 4 for determining the maximum amplitude $x\max$ of the elements of the vector X. Comparing means 5 compare this maximum value $x\max$ with a given reference value $xref$. If $x\max$ is below $xref$, switches 6 and 7 are opened, meaning that no correction algorithm is performed as the maximum value $x\max$ is below $xref$ which represents a maximum tolerable value for the amplitudes or values of the

vector X. In this case, the vector X is output via a subtractor 35 unchanged, since switch 7 supplying the negative input of subtractor 35 is opened.

If, however, $x\max$ exceeds $xref$, the switches 6 and 7 are closed. Via switch 6, the vector X is supplied to sorting means 8 which, with the help of the parameter M already described above, determines the auxiliary vectors Xm and Pm which are supplied to calculating means 9. Calculating means 9 performs the above-described iterative algorithm on the vector Xm and then computes the amplitude and phase values c_i and φ_i using the frequencies μ allocated for correction. At most L iterations are performed. If, however, $x\max$ falls below $xref$ before the L iterations are performed, the algorithm is terminated and the values for c_i and φ_i are output. In building means 10, the total correction vector Xk is built as described above and supplied via switch 7 to the subtractor 35 where it is subtracted from the vector X.

It should be noted in this respect that the vector representation serves as a means for easily representing the signals. However, the whole procedure may as well be viewed as carried out using the signals themselves, that is, emitting corresponding correction signals having the respective frequencies μ.

It is possible to use less frequencies μ for correcting the signals than actually allocated. For example, twelve frequencies may be allocated for correction, two or three of them may be used. These two or three frequencies should be changed from vector X to vector X, i.e. from frame to frame, to distribute the power of the correction signal over all correction frequencies.

Therefore, it is necessary to modify the algorithm slightly. If after the L-th iteration $x\max$ is still greater than $xref$, the procedure is repeated with a different choice of correction frequencies. If this is not the case after a certain number of tries, the frequencies which yielded the smallest value $x\max$ are used.

In the following, the performance of the method according to the invention will be demonstrated using simulation results.

Example 1: Downstream transmission in an ADSL system

For the inverse fast Fourier transform in ADSL systems generally 265 frequency values which are equally spaced from 0 to half the sampling frequency are defined. Therefore, a frame or vector X comprises 512 signal values, that is, N =512. The distance between carrier frequencies is 4.3125 KHz, resulting in a sampling frequency of 2.208 MHz. For data transmission the frequencies numbers 33 to 255 are used (142.3 to 1100 KHz). Two different sets of parameters were simulated. The first simulation was performed using frequency numbers 254, 217, 247, 225, 239, 231, 210 and 243 for correction purposes. M was set to 8, L, the maximum number of iterations, also to 8. $xref$ was set to 4.1. The power of the signal was normalized to 1, so that the peak value corresponds to the crest factor.

Figures 4A and 4B illustrates the results for these values. Figure 4A illustrates the probability p for the occurrence of various crest factors C given as a ratio, Figure 4B illustrates the same graph with the crest factor c given in decibel. Curve 11 shows the theoretical Gaussian distribution. Curve 12 shows the result without correction. The reason why curve 12 deviates from curve 11 is the limited simulation time, for a longer simulation time also the even higher crest factors would eventually occur. It can be seen that crest factors above 5.5 occur with a probability greater than 10^{-8}.

Curve 13 shows the results using the method according to the present invention. It can be seen that the probability for a crest factor of 4.1 or 12.25 dB is 10^{-8} corresponding to a reduction of 2.9 dB compared to the non-corrected case of curve 12.

For a second simulation, only five carrier frequencies were used for correction, namely numbers 240, 224, 208, 192 and 176. These five carrier frequencies are evenly spaced apart resulting in a periodic correction signal or correction vector Xk with a period of 32. M and L were both set to 8 as in the first simulation, and $xref$ was set to 4.3. The result as illustrated in Figures 5A and 5B corresponding to Figures 4A and 4B of the first simulation. Curve 11 again is the theoretical Gaussian distribution, curve 14 is the uncorrected curve corresponding to curve 12 of Figures 4A and 4B, and curve 15 is the corrected

curve using the method of the present invention. The deviations between curves 12 in Figures 4A and 4B and curves 14 in Figures 5A and 5B again stem from the statistical nature of the amplitude distribution and the limited simulation time. In this case, a crest factor of 4.4 corresponding to 12.85 dB occurs with a probability of 10^{-8}. Here, still a reduction of 2.3 dB compared to the uncorrected curve is obtained.

Consequently, it can be seen that the method of the present invention leads to a considerable reduction of the crest factor for ADSL transmissions.

Example 2: VDSL transmission

In VDSL systems 4096 frequency values are equally spaced apart from 0 to half the sampling frequency, resulting in a frame or vector X having 8192 values or elements. The distance between carrier frequencies is 4.3125 KHz corresponding to the ADSL value, resulting in a sampling frequency of 35.328 MHz.

For downstream and upstream transmission different frequency ranges are defined, which are illustrated in Figures 6A and 6B. Figure 6A illustrates the frequencies reserved for downstream transmission corresponding to frequencies number 257 to 695 and 1182 to 1634. Figure 6B illustrates the frequencies reserved for upstream transmission, that is, frequency numbers 696 to 1181 and 1635 to 2782.

First, a simulation for upstream transmission was performed.

Twelve possible carrier frequencies were allocated for correction purposes, frequencies number 2688, 2624, 2560, 2496, 2432, 2368, 2304, 2240, 2176, 2112, 2048, 1984. Three of these frequencies were used for actual correction. The allocated carrier frequencies are equally spaced apart and have a distance of 64 (i.e. 64×4.3125 KHz), resulting in a correction signal having a period of 128 independent of the actual choice of the three carrier frequencies used for correction. As parameters for the method of the present invention $M = 32$, $xref = 4.3$ and $L = 8$ were used. A maximum number of twelve choices of the three carrier frequencies were tried for each correction.

Figures 7A and 7B illustrates the simulation results, the representation of the results again being similar to those of Figures 4 and 5. Curve 11 again represents the theoretical Gaussian distribution, curve 16 the signal without correction and curve 17 the signal with correction according to the method of the present invention. A probability of 10^{-8} corresponds to a crest factor of 4.5 or 13 dB according to curve 17, which again is a considerable reduction compared to the 5.6 or 15 dB of the uncorrected curve 16.

For a downstream simulation, six carrier frequencies were used, namely frequencies number 1600, 1536, 1472, 1408, 1344 and 1280. The distance between the carrier frequencies again is 64, which again results in a periodic correction signal having a period of 128. For the simulation, the parameters $M = 32$, $xref = 4.3$ and $L = 8$ were used. 8A and 8B illustrate the results of this simulation. Curve 18 is the result without correction, curve 19 is the result with the correction according to the present invention. A probability of 10^{-8} corresponds to a crest factor of 4.65 or 13.4 dB, again yielding a significant improvement compared to the uncorrected signal.

It should be noted that the simulation examples given above are only given as an illustration, and other parameters may be used depending on the amount of crest factor reduction needed and the amount of computation time possible. For example, a larger value of M generally leads to a better reduction of the crest factor, but needs more calculation time. Other carrier frequencies than the one used in the simulations may be allocated for correction purposes.

Although exemplary embodiments of the invention are described above in detail, this does not limit the scope of the invention, which can be practiced in a variety of embodiments.